Módulo 13

1. b

f_1 (x): exponencial de base 10 (gráfico 2); f_2(x); logarítmica de base 10 (gráfico 4);

f_3(x) = x: função identidade (= composição de f_1 com sua inversa, $f_2 \Rightarrow$ gráfico 1);

f_4(x) = 2x+1 $\Rightarrow$ gráfico 3.

2. b

$$2^x = 5 \Rightarrow x = \log_2 5 = \frac{\log 5}{\log 2}$$

3. e

$$\frac{1}{\log_x 3} + \frac{1}{\log_{\sqrt{x}} 3} + \frac{1}{\log_{\sqrt[4]{x}} 3} + \frac{1}{\log_{\sqrt[8]{x}} 3} = \frac{15}{8} \Rightarrow \log_3 x + \log_3 \sqrt{x} + \log_3 \sqrt[4]{x} + \log_3 \sqrt[8]{x} = \frac{15}{8}$$

$$\log_3\left(x.\sqrt{x}.\sqrt[4]{x}.\sqrt[8]{x}\right) = \frac{15}{8} \Rightarrow \log_3\left(x.x^{\frac{1}{2}}.x^{\frac{1}{4}}.x^{\frac{1}{8}}\right) = \frac{15}{8} \Rightarrow \log_3\left(x^{\frac{15}{8}}\right) = \frac{15}{8} \Rightarrow 3^{\frac{15}{8}} = x^{\frac{15}{8}} \Rightarrow x = 3$$

$$\log_3 x = \log_3 3 = 1$$

4. e

$$\left(\frac{1}{2}\right)^{\log_2 x} < \left(\frac{1}{2}\right)^3 \Rightarrow \log_2 x > 3 \Rightarrow x > 2^3 \Rightarrow x > 8$$

A condição de existência é: $x > 0$. Da intersecção, resulta: $S = \{x \in \mathrm{IR} \mid x > 8\}$

5. d

$$\log_{\frac{1}{3}}\left(\log_{\frac{1}{3}} x\right) \geq 0 \Rightarrow \log_{\frac{1}{3}} x \leq 1 \Rightarrow x \geq \frac{1}{3}$$

$$\text{C.E.}: x > 0 \text{ e } \log_{\frac{1}{3}} x > 0 \Rightarrow x < 1$$

Das intersecções, resulta: $S = \left\{x \in \mathrm{IR} \mid \frac{1}{3} \leq x < 1\right\}$

6. b

$$r_1 - r_2 = \log_{10}\left(\frac{m_1}{m_2}\right) \Rightarrow 5{,}9 - 5{,}8 = \log_{10}\left(\frac{m_1}{m_2}\right) \Rightarrow \frac{m_1}{m_2} = 10^{0{,}1} = 10^{\frac{1}{10}} = \sqrt[10]{10}$$

7. a

$$pH = -\log\left[H^+\right] \Rightarrow pH = -\log\left(5{,}4.10^{-8}\right) = -\left(\log 5{,}4 + \log 10^{-8}\right) = -\log\left(\frac{2.3^3}{10}\right) - (-8) =$$

$$-\left(\log 2 + 3.\log 3 - \log 10\right) + 8 = -\left(0{,}3 + 3.0{,}48 - 1\right) + 8 \Rightarrow pH = 7{,}26$$

8.

$$9^{2x+1} = 45 \Rightarrow \log\left(9^{2x+1}\right) = \log\left(3^2.5\right) \Rightarrow (2x+1).\log 9 = \log\left(3^2\right) + \log 5 \Rightarrow 2x+1 = \frac{2\log 3 + \log 5}{\log\left(3^2\right)}$$

$$2x+1 = \frac{2\log 3 + \log 5}{2\log 3} = \frac{2.0{,}5 + \log\left(\frac{10}{2}\right)}{2.0{,}5} = \frac{1 + \log 10 - \log 2}{1} = 1 + 1 - 0{,}3 = 1{,}7$$

$$2x = 0{,}7 \Rightarrow x = 0{,}35 = \frac{35}{100} = \frac{7}{20}$$

$$S = \left\{\frac{7}{20}\right\}$$

9.

$$V = V_0.(1+i)^t \Rightarrow 3.V_0 = V_0.(1+0{,}09)^t \Rightarrow 3 = 1{,}09^t \Rightarrow \ln 3 = \ln\left(1{,}09^t\right) \Rightarrow t.\ln 1{,}09 = \ln 3$$

$$t = \frac{\ln 3}{\ln 1{,}09} = \frac{1{,}08}{0{,}09} \Rightarrow t = 12 \text{ meses}$$

10.

$$R = 12 + \log_{10} I$$

(1)V

$$0 = 12 + \log_{10} I \Rightarrow \log_{10} I = -12 \Rightarrow I = 10^{-12}\ W/m^2$$

(2)F

$$R_{avião} = 16\,bels \Rightarrow 16 = 12 + \log_{10} I \Rightarrow I = 10^{4}\ W/m^2$$

$$R_{tráfego} = 8\,bels \Rightarrow 8 = 12 + \log_{10} I \Rightarrow I = 10^{-4}\ W/m^2$$

(3)V (conforme calculado em (2)).

11.

$$f(x) = 2^{2x} \Rightarrow f(0) = 2^0 = 1;\ f(1) = 2^2 = 4;\ g(1) = \log_2 2 = 1$$

As coordenadas dos vértices do triângulo são os pontos: $A(1;1), B(0;1)$ e $C(1;4)$.

A área do triângulo é dada por: $A_\Delta = \frac{1}{2}.\left\|\begin{matrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 4 & 1 \end{matrix}\right\| = \frac{3}{2}\ u.a.$

12.

$$f(x)=\log_2(k.x), k>0$$

$$f(2)=0 \Rightarrow \log_2(2k)=0 \Rightarrow 2k=1 \Rightarrow k=\frac{1}{2} \Rightarrow f(x)=\log_2\left(\frac{x}{2}\right)$$

$$f(4)=\log_2\left(\frac{4}{2}\right)=\log_2 2=1 \Rightarrow A_{retângulo}=4\times1=4\ u.a.$$

$$f(x)=-2 \Rightarrow \log_2\left(\frac{x}{2}\right)=-2 \Rightarrow x=\frac{1}{2} \Rightarrow A_{trapézio}=\frac{\left(4+\frac{1}{2}\right)\times 2}{2}=4{,}5\ u.a \Rightarrow A_{total}=4+4{,}5=8{,}5\ u.a.$$

13.

$$V=V_0.(1+i)^t \Rightarrow 5.V_0=V_0.(1+i)^t \Rightarrow 1{,}25^t=5 \Rightarrow \log\left(1{,}25^t\right)=\log 5 \Rightarrow t.\log 1{,}25=\log 5$$

$$t.\log\left(\frac{10}{8}\right)=\log\left(\frac{10}{2}\right) \Rightarrow t.(\log 10-\log 8)=\log 10-\log 2 \Rightarrow t.\left(1-\log\left(2^3\right)\right)=1-0{,}30103$$

$$t.(1-3.\log 2)=0{,}69897 \Rightarrow t.(1-3.0{,}30103)=0{,}69897 \Rightarrow t=7{,}212 \text{ anos}$$

$$\therefore t_{mín}=8 \text{ anos}$$

14.

$$V=V_0.(1+i)^t \Rightarrow 2.V_0=V_0.(1+0{,}12)^t \Rightarrow 2=1{,}12^t \Rightarrow \log 2=\log\left(1{,}12^t\right) \Rightarrow t.\log 1{,}12=\log 2$$

$$t.\log\left(\frac{2^4.7}{100}\right)=0{,}30 \Rightarrow t.\left(\log\left(2^4\right)+\log 7-\log 100\right)=0{,}30 \Rightarrow t.(4.\log 2+\log 7-\log 100)=0{,}30$$

$$t.(4.0{,}30+0{,}84-2)=0{,}30 \Rightarrow t=7{,}5 \text{ anos}=7 \text{ anos e } 6 \text{ meses}$$

15.

a) $P(t)=P_0.2^{-bt} \Rightarrow P(29)=P_0.2^{-b.29} \Rightarrow \frac{P_0}{2}=P_0.2^{-29b} \Rightarrow 2^{-1}=2^{-29b} \Rightarrow 29b=1 \Rightarrow b=\frac{1}{29}$

b)

$$P(t)=P_0.2^{-\frac{1}{29}t} \Rightarrow \frac{P_0}{5}=P_0.2^{-\frac{t}{29}} \Rightarrow \log_2\left(\frac{1}{5}\right)=\log_2\left(2^{-\frac{t}{29}}\right) \Rightarrow \log_2 1-\log_2 5=-\left(\frac{t}{29}.\log_2 2\right)$$

$$0-\log_2\left(\frac{10}{2}\right)=-\frac{t}{29}.1 \Rightarrow 0-(\log_2 10-\log_2 2)=-\frac{t}{29} \Rightarrow 0-(3{,}32-1)=-\frac{t}{29} \Rightarrow t=67{,}28 \text{ anos}$$