Módulo 5

5.

Fazendo-se a projeção da figura num plano que contém o triângulo, tem-se uma circunferência inscrita neste triângulo.

$$A_{\Delta} = p.r \Rightarrow \frac{l^2\sqrt{3}}{4} = \left(\frac{3l}{2}\right).r \Rightarrow r = \frac{l\sqrt{3}}{6}$$

$$V_{prisma} = A_{base}.H = \left(\frac{l^2\sqrt{3}}{4}\right).(2r) = \left(\frac{l^2\sqrt{3}}{4}\right).\left(2.\frac{l\sqrt{3}}{6}\right) = \frac{l^3}{4}$$

$$V_{esf} = \frac{4}{3}\pi r^3 = \frac{4}{3}.3.\left(\frac{l\sqrt{3}}{6}\right)^3 = \frac{l^3\sqrt{3}}{18}$$

$$\frac{V_{esf}}{V_{prisma}} = \frac{\left(\frac{l^3\sqrt{3}}{18}\right)}{\left(\frac{l^3}{4}\right)} \cong 0,38 = 38\%$$

6.

a)

$$A_L = 2.(15.40 + 15.40) = 2400\,cm^2$$

$$A_{\sup} = 4\pi R^2 \Rightarrow A_{\sup} = 4.3.2^2 = 48\,cm^2$$

b)

$$V_{rec} = 15.15.40 = 9000\,cm^3$$

$$V_{esf} = \frac{4}{3}\pi.2^3 = 32\,cm^3$$

$$V_{líq} = 9000 - (90.32) = 6120\,cm^3$$

7.

$$g^2 = h^2 + r^2 \Rightarrow 2,5^2 = 2^2 + r^2 \Rightarrow r = 1,5\,m$$

$$V_{cone} = \frac{1}{3}.(\pi.1,5^2).2 = 4,5\,m^3$$

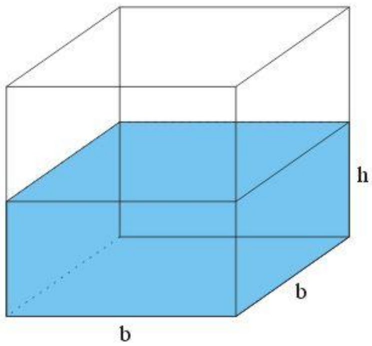
$$\frac{v}{V} = \left(\frac{h'}{h}\right)^3 \Rightarrow \frac{v}{4,5} = \left(\frac{1}{2}\right)^3 \Rightarrow v = 0,5625\,m^3$$

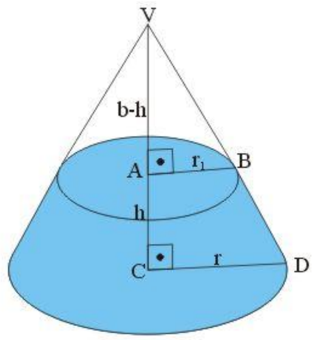
$$\Delta V = 4,5 - 0,5625 = 3,9375\,m^3$$

8.

Como os tanques têm a mesma capacidade e a mesma altura, fica:

$$b^3 = \frac{\pi r^2 b}{3} \Rightarrow r^2 = \frac{3b^2}{\pi}$$





Os triângulos retângulos VAB e VCD são semelhantes, logo:

$$\frac{b-h}{r_1} = \frac{b}{r} \Rightarrow r_1 = \frac{r(b-h)}{b}$$

$$V_2(h) = \frac{\pi r^2 b}{3} - \frac{(b-h).\pi}{3}.\left[\frac{r.(b-h)}{b\pi}\right]^2 = \frac{\pi r^2 b^3 - \pi r^2 (b-h)^3}{3b^2} = \frac{\pi r^2}{3b^2}.\left(3b^2 h - 3bh^2 + h^3\right)$$

$$r^2 = \frac{3b^2}{\pi} \Rightarrow V_2(h) = \frac{3b^2}{\pi}.\frac{\pi.\left(3b^2 h - 3bh^2 + h^3\right)}{3b^2} = 3b^2 h - 3bh^2 + h^3$$

$$V_{água} = V_1(h) = b^2 h$$

$$V_2(h) = 3.V_1(h) \Rightarrow 3b^2 h - 3bh^2 + h^3 = 3b^2 h \Rightarrow h^3 = 3bh^2 \Rightarrow h = 3b \Rightarrow \frac{h}{b} = 3$$

9.

a) $A = \pi.\left(3^2 - 1^2\right) = 8\pi\, cm^2$

b) $V = \frac{4}{3}\pi\left(r'\right)^3 = \frac{4}{3}\pi.2^3 = \frac{32\pi}{3} cm^3$

10.

Ligando os centros das circunferências, pode-se construir um triângulo retângulo em seguida.

Aplicando o teorema de Pitágoras: $13^2 = 5^2 + y^2 \Rightarrow y = 12\,cm$

$$H = y + R + r \Rightarrow H = 12 + 8 + 5 = 25\,cm \Rightarrow V_{cilindro} = \pi.9^2.25 = 2025\pi\, cm^3$$

$$V_{esf\,1} = \frac{4}{3}\pi.8^3 = \frac{2048\pi}{3} cm^3$$

$$V_{esf\,2} = \frac{4}{3}\pi.5^3 = \frac{500\pi}{3} cm^3$$

$$V_{final} = 2025\pi - \left(\frac{2048\pi}{3} + \frac{500\pi}{3}\right) = \frac{3527\pi}{3} cm^3$$

$$h_{final} = \frac{V_{final}}{A_{base}} = \frac{\left(\frac{3527\pi}{3}\right)}{\pi.9^2} \Rightarrow h_{final} \cong 14,5\,cm$$